# THE DATE OF ANONYMUS HEIBERG,
# ANONYMI LOGICA ET QUADRIVIUM.

### C.M. Taisbak.

I

In his study of "Commentators and Commentaries on Aristotle's Sophistici
Elenchi" Dr. Sten Ebbesen writes, vol. I p. 262:

"In the autumn of 1007 a learned Greek was writing a compendium
of astronomy. In a manuscript from 1040 - and several later
ones - this opuscule is the last of four which make up a compen-
dium of the quadrivium that is part two of a work whose part one
is a compendium of logic, or "Survey of Philosophy" (Συνοπτικὸν
σύνταγμα φιλοσοφίας) according to its title. The compendia of
logic and the quadrivium were edited in 1929 by J.L.Heiberg, after
whom I call the author (and his work) 'Anonymus Heiberg'."

Below is reproduced part of the astronomical compendium, p. 107,25 - 109,18,
from which the following facts emerge: The actual year is stated twice
(108,14 and 109,10) to be 6516. We shall understand that as referring to
the Byzantine world era, accepting the equation

(11)         year 1 Byzantine SEP 1 = year -5508 Julian SEP 1,

- the Byzantine new year beginning with September, analogously to the Alex-
andrian year, which we shall discuss below.[1] Thus the year of the text,
6516, must begin with 1007 Julian SEP 1 and run till 1008 AUG 31. This
dating is corroborated by the procedure which the text describes for con-
verting an Egyptian date into a Julian one, with a view to applying tables
for computing positions of the sun and the moon (presumably Ptolemy's Handy
Tables[2]). We shall premise a few well-known facts.

1    The tropical year (which is the time interval between two consecutive
vernal equinoxes) is 365 days 5 hours 48 minutes 46 seconds, that is 11
minutes 14 seconds shorter than 365 + 1/4 days.

2    The Julian Year (which was established by C. Julius Caesar from Jan-
uary -44 (= 45 B.C.)) is 365 + 1/4 days, so that three consecutive years
have 365 days, whereas the fourth (namely the years -40, -36, -32, ..., -4,

---

1. For the Byzantine era, cf. GINZEL III 292. I do not know why NEUGEBAUER
III 1066 gives the year 1 Byzantine as -5507.
2. The tables are mentioned twice on p.109 as Πτολεμαίου Κανόνιον and ac-
cording to their description 109,8 ff. must be his Πτόχειροι Κανόνες,
"Handy Tables". They are being edited by ANNE THON, together with Theons
Great Commentary to them.

0, 4, 8, ...) have 366 days, – February 24, *ante diem sextum Kal. Martias*,
being counted twice, *bisextum*. For every practical purpose the Julian
year is equal to the tropical. We shall adopt the modern convention of
projecting the Julian years backwards before their introduction by Julius
Caesar, and apply zero and negative numbers for years Before Christ; thus
1 A.D.= 1, 1 B.C.= 0, 2 B.C.= -1, etc. This convention is not ideological,
but serves to facilitate subtraction.[3]

3    The Egyptian year is a "wandering" year, in the following sense: Each
Egyptian year has 365 days, namely 12 months of 30 days and 5 "added" days,
*epagómenai*. It is 6 hours shorter than the Julian year, which appears in
the following way: If the first day (Thoth 1) of the Egyptian year falls
on, say, the year 139 JULY 21, then in 140 (which has *bisextum*, is a leap
year) it will fall on JULY 20, and in the following years 141, 142, and
143 it will still fall on that date; but in 144 (leap year) it will "ad-
vance" to JULY 19, and thus the New Year's Day of the Egyptian year will
"wander" through the Julian and the tropical year, that is: through the
seasons. In 1460 years it will wander through a whole year, so that 1460
Julian years = 1461 Egyptian years.

4    The Alexandrian year is identical with the Egyptian, with the one ex-
ception that every fourth year has 6 and not 5 *epagomenai*. It was intro-
duced by Augustus in the eastern provinces in the fifth year of his reign,
counting from the battle of Actium. Thus

(12)              year 5 Augusti Thoth 1 = -25 Julian AUG 30.

The Alexandrian year is fixed in relation to the Julian; the 6th *epagomene*
occurs in August of the years before Julian leap years.

5    Since Ptolemy's astronomical tables use Egyptian years throughout,
astronomers and astrologers must be able to convert Julian and Alexandrian
dates into Egyptian ones. Egyptian years are counted by the era of Philip
Arridaius, from the death of Alexander the Great, so that

(13)              year 1 Philippi Thoth 1 = -323 Julian NOV 12.

The three eras are connected by the equation

(12a) 5 Augusti Thoth 1 = 299 Philippi Thoth 1 = -25 Julian AUG 30.

6    The text (107.27 - 108.12) establishes a concordance between Egyptian
and Julian months, so that Thoth corresponds to September, Phaophi to Oc-
tober etc.; (the names of the Egyptian months can be learnt from 108.19-20;
beware of the "month" *epagómenos*, which is nothing but the 5 or 6 *epagómenai*,

---

3. NEUGEBAUER discusses the problem III 1061-1062.

"added" days). However, Thoth begins 3 days earlier than September[4], whence Thoth 1 =AUG 29. If you remember which Julian months have 31 days, and if you give February a special treatment, considering whether it be leap year or not, you will find it easy to convert Julian dates into Egyptian ones, and vice versa.

7     But then Thoth 1 does *not* coincide with AUG 29. It did so when the Alexandrian calendar was new, 6 Augusti Thoth 1 beeing −24 AUG 29 (cf.12a); but the first Alexandrian "leap day", 8 Augusti Epag 6 (= −21 AUG 29) coincides with 303 Philippi Thoth 1, and from then the Egyptian New-Year's Day advances in relation to the Alexandrian one, 1 day every 4 years, according to the formula

(14)      9 Augusti Thoth 1 =     303 Philippi Thoth 2,
         13    −    Thoth 1 =     307    −    Thoth 3,
        $4n + 5$   −   Thoth 1 = $4n + 299$   −   Thoth 1 + n.

It becomes a separate task to account for the number n, which is the number of days by which the Egyptian Thoth 1 precedes the Alexandrian Thoth 1[5]. The text speaks of adding the *tetraeteridas*, the 4-years (108.12-13), and instructs us to add 258[6]. That is to say: the Egyptian year in question precedes the Alexandrian year by 258 days; and from formula (14) we compute the years for n = 258:

     1037 Augusti Thoth 1 = 1331 Philippi Thoth 259,
which date is 1331 Philippi Pachon 19, Pachon being the ninth month of the year. If the number 258 is correct, the actual year (according to formula (14)) must be one of the four years following 1037 Augusti Thoth 1.

8     And it appears to be correct, for the year of era Philippi corresponding to 1037 Augusti, namely 1331, is given in the text (109.9-10):   In Ptolemy's Handy Tables the positions of the sun and the moon are given in periods of 25 years (because 25 years very nearly equal 309 synodic months), for the years 1, 26, 51, 76, ...25m + 1 Philippi, ...   Our text says that

---

4. "One must take the three days which surpass the thirty in the months of May, July and August", 108,1-2. This nonsense must be understood as a mnemotechnical aid; at least, I see no special reasons for involving the said months in the fixation of Thoth 1.

5. A minor work of Theon's, commented on by NEUGEBAUER II 966, gives those ἐπακταὶ ἤτοι ἐμβόλιμοι, "epacts or intercalary days", for the years A.D. 138 to 372.

6. The text erroneously believes that the tetraeterides are counted from the reign of Philip; but of course, the counting of 4-year-periods plays no rôle before the fixation of the Alexandrian Thoth 1. The phrase of 108,13-14 would be appropriate in 109,9; evidently the author did not know why it all worked, cf. note 4.

"we take the periods-of-25-years from the table, which are for the actual year 1326 plus 5 single years". Thus 1325 + 5 years have elapsed since the beginning of era Philippi, and we are in 1331 Philippi.

9    Now, the number n = 258 (in formula (14)) is valid only from 1037 Augusti Thoth 1, the year 1036 Augusti being a leap year with a 6th *epagomene*; which means, if 1331 is to be "saved", that the text was written some time

between 1331 Philippi Pachon 19 and Epagomene 5.

Converted into Alexandrian dates the interval falls

between 1037 Augusti Thoth 1 and Choiac 17,

Choiac being the fourth month of the year.

The corresponding Julian dates, computed from equation (12), are

between 1007 AUG 30 and DEC 14.

Since the year, in the Byzantine era, is numbered 6516, which according to equation (11) begins 1007 Julian SEP 1, we may conclude that if the numbers of the text are correct and correctly used by the author, it was written some day between 1007 SEP 1 and DEC 14. Which was to be proven.

*Sources:*

ANONYMI Logica et Quadrivium, cum Scholiis Antiquis edidit J.L. HEIBERG (Det Kgl.Danske Videnskabernes Selskab, Historisk-filologiske Meddelelser XI,1) København 1921.

EBBESEN, S. Commentators and Commentaries on Aristotle's Sophistici Elenchi. 3 vols. (Brill) Leiden 1981.

GINZEL, F.K. Handbuch der mathematischen und technischen Chronologie. 3 vols. Leipzig 1906-14 (reprinted 1958).

NEUGEBAUER, O. A History of Ancient Mathematical Astronomy. 3 vols. (Springer) Berlin, Heidelberg, New York 1975.

*Appendix:*

Pp. 107.25-109.18 of Heiberg's 1929 edition, reprinted with the kind permission of the Royal Danish Academy of Sciences and Letters.

25     ἀλλὰ γὰρ ἀρκτέον ἐντεῦθεν τῆς ψηφοφορίας ἡμῖν εὐσύνοπ- 8
τον τὸν περὶ αὐτῆς διεξιοῦσι λόγον.

δεῖ πρὸ μηνὸς Σεπτεμβρίου τοῦ παρ' Αἰγυπτίοις λεγομένου

---

**108**           Nr. 1. J. L. HEIBERG:

Θώθ, ἀφ' οὗ τὰ τῆς ψηφοφορίας ἄρχεται, τρεῖς λαμβάνειν
ἡμέρας τὰς ὑπὲρ τὴν τριακάδα Μαΐου, Ἰουλίου καὶ Αὐγούστου
καὶ ταύταις ταῖς τρισὶ προστιθέναι τὰς ὑπὲρ τὴν τριακάδα
πάλιν ἡμέρας τῶν μετὰ Σεπτέμβριον μηνῶν, ἕως ἂν καταν-
τήσωμεν εἰς αὐτὸν τὸν μῆνα, καθ' ὃν τὴν ψηφοφορίαν ποιού- 5
μεθα, καὶ ἀφιέναι ὑπὲρ τοῦ Φεβρουαρίου δύο ἡμέρας ἢ μίαν,
ἐὰν ᾖ βίσεξτον, καὶ τὰς λοιπὰς συνάπτειν ταῖς τοῦ μηνὸς
ἡμέραις, ὅσας ἂν τύχῃ ἔχων, καθ' ἣν ψηφοφοροῦμεν ἡμέραν,
καὶ τὰς συναθροιζομένας ἡμέρας τὸν αὐτὸν ἔχειν μῆνα παρ'
Ἀλεξανδρεῦσί φαμεν. εἰ δὲ ὑπερτείνει τὰς λ̅ ἡμέρας ὁ ἀριθ- 10
μός, τριακοντάδα ἐξαιροῦντες τὰς λοιπὰς εἶναί φαμεν τοῦ
ἐφεξῆς παρ' Ἀλεξανδρεῦσι μηνός, αἷς προστιθέντες τὰς τετρα-
ετερίδας, αἵτινές εἰσιν ἀπὸ τῆς βασιλείας ψηφιζόμεναι Φιλίπ-
που ἄχρι τοῦ δεῦρο ἤγουν τοῦ ‚ϛφιϛ' ἔτους σ̅ν̅η̅, καὶ ἀπὸ τῶν
ἐπισυναχθέντων ἐκβάλλοντες καθεξῆς ἀπὸ τοῦ παρ' Ἀλεξαν- 15
δρεῦσι μηνὸς ὑπὲρ ἑκάστου μηνὸς ἀνὰ λ̅ ἡμέρας, εἰς οἷον ἂν
καταντήσῃ μῆνα τὸ πέρας τοῦ ἀριθμοῦ, ἐκεῖνον εἶναι φήσομεν
Αἰγύπτιον μῆνα τὰς ὑπολειφθείσας ἡμέρας ἐπέχοντα. οἱ δὲ
Αἰγύπτιοι μῆνές εἰσιν οὗτοι· Θώθ, Φαωφί, Ἀθύρ, Χοιάκ, Τυβί,
Μεχείρ, Φαμενώθ, Φαρμουθί, Παχών, Παῦνί, Ἐπιφί, Μεσωρί, 20
ἐπαγόμενος, ὅστις καὶ πενθήμερός ἐστιν ἐν τρισὶν ἔτεσιν, ἐν
δὲ τῷ τετάρτῳ τῷ καὶ βισέξτῳ ἑξήμερος.

**9**    εὑρίσκοντες οὖν οὕτω τὸν Αἰγύπτιον μῆνα κρατοῦμεν καὶ
τὴν ὥραν κατ᾽ αὐτὴν τὴν ἡμέραν τῆς ψηφοφορίας ἀπὸ μεσημ-
βρίας ἀρχόμενοι πρώτην τὴν ἑβδόμην ὥραν τῆς ἡμέρας τιθέντες 25
κἀκεῖθεν ἀριθμοῦντες μέχρις αὐτῆς τῆς ὥρας, καθ᾽ ἣν ποιού-
μεθα τὴν ψηφοφορίαν, εἴτε ἡμερινὴ εἴη εἴτε νυκτερινὴ εἴτε

τῆς ἐπιούσης ἡμέρας, εἰ τυχὸν εἰς αὐτὴν ἐμπέσοι ὁ ἀριθμός,
ὡς εἶναι φέρε εἰπεῖν ἐννεακαιδεκάτην ὥραν εἴτε καὶ εἰκο-
στὴν καὶ ἐφεξῆς μέχρις εἰκάδος τετάρτης, ἥτις ἐστὶν ἕκτη
τῆς ἐπιούσης ἡμέρας. προστίθεμεν δὲ τῇ ἐν τῇ Βυζαντίδι λαμ-
5 βανομένῃ ὥρᾳ καὶ τὴν τῆς Ἀλεξανδρείας πρὸς τὸ Βυζάν-
τιον τῆς ὥρας ὑπεροχὴν λεπτὰ $\overline{μη}$ $\overline{μ}$ διὰ τὸ ἀνατολικωτέραν
εἶναι τοῦ Βυζαντίου τὴν Ἀλεξάνδρειαν καὶ πρὸς αὐτὴν ἐκκεῖ-
σθαι τὸ τοῦ Πτολεμαίου κανόνιον. λαμβάνομεν δὲ καὶ τὰς
εἰκοσιπενταετηρίδας ἀπὸ τοῦ κανονίου, αἵ εἰσι κατὰ τὸ νῦν
10 ,ϛφιϛ´ ἔτος ἔτη ,ατκϛ $\overline{ἁπλᾶ}$ ἔτη πέντε, καὶ οὕτω λοιπὸν εὑρί-
σκοντες τὸν Αἰγύπτιον μῆνα, τὴν ἡμέραν, τὴν ὥραν, τὰς εἰκοσι-
πενταετηρίδας καὶ τὰ ἁπλᾶ ἔτη σκοποῦμεν ἐπὶ μὲν τοῦ ἡλίου
τὰς παρακειμένας τοῖς αὐτοῖς πέντε κεφαλαίοις ἐν τῷ κανο-
νίῳ τοῦ Πτολεμαίου κατὰ τὸ ἀπόγειον τοῦ ἡλίου μοίρας καὶ
15 τὰ λεπτὰ καὶ τὰς ἑξηκοντάδας τῶν λεπτῶν εἰς μοίρας ἀναβι-
βάζοντες καὶ ταῖς μοίραις προστιθέντες καὶ πάντα ὁμοῦ συγ-
κεφαλαιοῦντες ἀφαιροῦμεν ἐξ ·αὐτῶν κύκλον ἢ κύκλους, εἰ πλεί-
ους σώζοιεν κύκλους ἑκάστῳ κύκλῳ ἀνὰ $\overline{τξ}$ μοίρας νέμοντες